Using Quantum Programming Languages for Cryptanalysis

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Overview

In this talk I'll showcase 'non-asymptotic' cryptanalysis. I will cover:

- The crypto setting,
- Applications,
- An example work on AES using Q#,
- What features were the most useful so far, and what new useful features could be.

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Disclaimer

My personal experience covers mostly Q#, with very little use of Qiskit and ProjectQ.

Cryptography

The study of techniques to provide confidentiality, integrity and authenticity to communications.

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- Cryptographic functionality gets its security from computationally "hard" problems.
- Cryptanalysis allows us to determine how small we can choose problem instances, while achieving a target security level.

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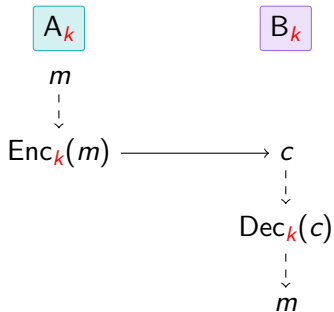
Block ciphers (eg. AES)

Two parties share an identical *key* k.



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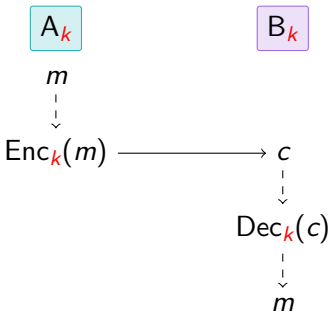
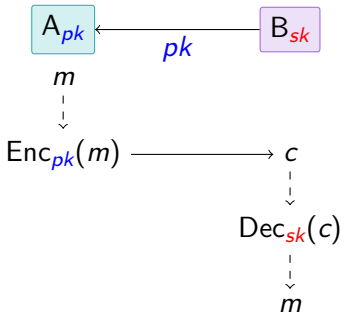


Public-key encryption (eg. RSA)

Different keys for sender and receiver.



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Polynomial time algorithm for solving RSA.

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We will now look at a non-asymptotic Grover scenario, using AES as an example.

(N, M)-unstructured search problem

Given a randomly sorted list L of size N and a property P such that exactly M elements of L satisfy P, find one such element.

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AES block cipher

Block cipher with encryption function E: $\{0,1\}^k \times \{0,1\}^n \to \{0,1\}^n$.
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Attacking AES: given (m, c), find k such that $c \leftarrow E(k, m)$.

Since $E(\cdot, m) \sim \$$, this is an unstructured search in $\{0,1\}^k$.

$\implies$ Classical runtime $\approx 2^k$ encryptions, one per key (trivially parallelisable).

$\implies$ Quantum runtime $\approx 2^{k/2}$ Grover steps (badly parallelisable).

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- General reason: doubling keys may be practically inconvenient (and overkill).
- Particular reason: the hardness of AES is being used as a definition of security.

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NIST Post-Quantum Cryptography standardisation

- Since 2017, the US NIST has been running a process to standardise post-quantum public-key cryptographic schemes.
- To qualify for "category 5" security, a scheme should be as secure as AES-256.
- Extra constraint: upper bound `MAXDEPTH` on the depth of quantum computation.

Post-quantum cryptography

NIST wants to standardise new *public-key encryption* and *signature schemes* with:

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- Well-understood hardness against quantum adversaries,
- Smallest possible size.

A practical analysis of Grover's algorithm is also useful here.

- Most quantum attacks on the proposals are classical attacks using Grover speed-ups for searching steps.

NIST concerns attacking AES-256

Parallelisation

NIST considers a hard limit $\texttt{MAXDEPTH} \in \{2^{40}, 2^{64}, 2^{96}\}$.
$\texttt{MAXDEPTH} < 2^{k/2} = 2^{128}$, what is naively required by Grover's
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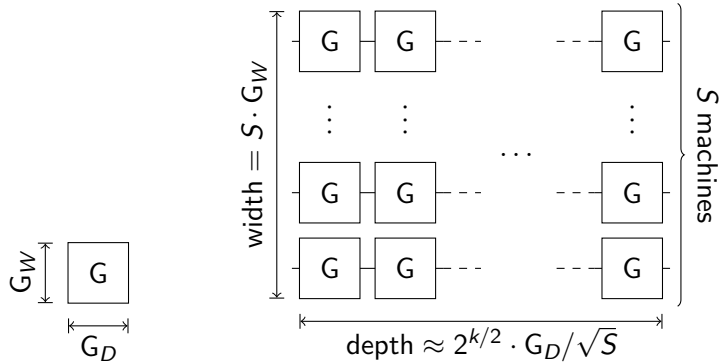
Resource estimation

NIST further proposes the measure of security being the number of quantum gates required for the attack.
$\implies$ How many gates is $O(2^{k/2})$ Grover steps?

Parallel Grover

Grover parallelises badly [Zal99]. Rule of thumb, using S machines gives a $\sqrt{S}$ speed-up. For a fully analysis in our setting, see [JNRV20].



(a) Grover step (b) Parallel Grover circuit

$\Longrightarrow$ To derive S we need to know the size of G.

Resource estimation

The main component of G is U_f: $|x\rangle\,|\varphi\rangle \mapsto (-1)^{f(x)}\,|x\rangle\,|\varphi\rangle$, where f marks solutions to the search problem.

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- G's width: the number of qubits used,
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Strategy

Implement G (hence AES) in Q#, use the resource estimator to find circuit size.

Advantages vs manual circuit design

- partially unit-testable,
- friendly to read/modify,
- automated circuit size estimates,
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Let's now look at the components of AES.

- AES is a substitution-permutation network (SPN).
- Two parallel processes, one updates a round key, the other operates on the message block. Both operate in repeated *rounds*.

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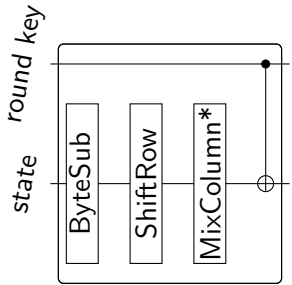

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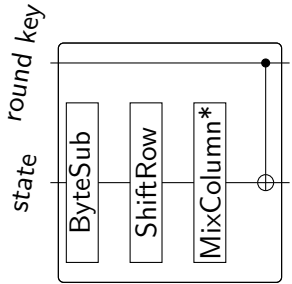
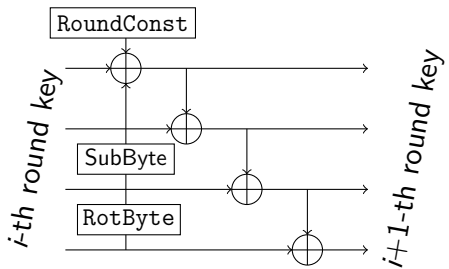


(g) AES encryption round



(h) AES (-128) key expansion round

Credit for key expansion: Jérémy Jean.

Component	Mathematical operation	Quantum gates
ShiftRow	Permutation	Rewiring
RotByte	Permutation	Rewiring
RoundConst	Fixed bit-flip	X
MixColumn	Invertible affine map $\mathbb{F}_2^{32}$	CNOT
SubByte, ByteSub	Inversion + affine maps over $\mathbb{F}_{2^8}$	NOT, XOR, AND/CCNOT

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There are a few options for generating linear and non-linear maps:

- LU matrix decomposition allows in-place invertible maps, no need for ancillas,
- Porting linear programs from the classical hardware design by implementing $(x, y, z) \mapsto (x, y, z \oplus (x \star y))$ for any binary operation $\star$,
- Using SAT-based synthetisers (eg. [FSK10, MSR+19, MSC+19]).

There is also room for choosing different binary operation circuits. For example, we can use AND instead of CCNOT.

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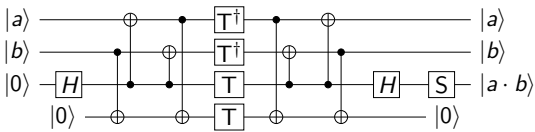
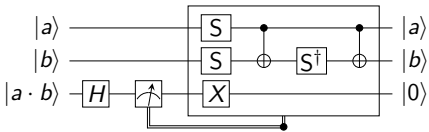


(k) AND gate.



(l) AND† gate.

We use a combination of Selinger's [Sel13] and Jones' [Jon13] circuits to express the AND gate in terms of Clifford and T gates.

$\Longrightarrow$ It requires expressing conditional branching on measurement outputs.

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We said asymptotically the quantum cost is $\approx 2^{128}$, the classical cost $\approx 2^{256}$.

MAXDEPTH	# of parallel machines	# of gates
2^{40}	$2^{197.15}$	$2^{245.46}$
2^{64}	$2^{149.15}$	$2^{221.46}$
2^{96}	$2^{85.17}$	$2^{190.47}$

Table: Derived gate costs for attacking AES-256 using Grover's algorithm.

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Table: Derived gate costs for attacking AES-256 using Grover's algorithm.

$\Longrightarrow$ Doubling keys may not be necessary.
$\Longrightarrow$ Quantum speed-ups with depth limit not as dramatic for symmetric crypto.

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We've seen one approach to non-asymptotic quantum cryptanalysis.

- How popular is it? What languages are commonly used?
- What language features are or would be useful?

Q#	[JNRV20] AES
	[HJN+20] Shor's on elliptic curves
	[BJ20] Simon's against various block ciphers
	[DP21] Search With Two Oracles technique for AES
Qiskit	[AMM20a] SIMON
	[CS20] ARIA
	[Sch20] Gimli
	[AMM+20b] FSR-based constructions
ProjectQ	[LPS20] AES S-box
	[JCKS20] SPECK
	[JCK+20] Korean block ciphers
IBMQ	[MSS21] Simon's algorithm (on `ibmq_16_melbourne`)
Other	[GLRS16] AES (in C)
	[RNSL17] Shor's on elliptic curves (in LIQUi|⟩)
	[AGPS20] Nearest-Neighbour Search for lattice sieving (in Python)

Table: Selection of works implementing cryptanalytic quantum algorithms.

Quantum Period Finding on IBM-Q

May *et al.* run period finding on $\mathbb{F}_2^7 \mapsto \mathbb{F}_2^7$ using `ibmq_16_melbourne`.

Approach

- Manually optimise circuit compatible with computer's topology.
- Apply various noise-smoothing techniques to address qubit quality and "bias towards 0".
- Recovering the period on NISQ is reduced to Learning Parity with Noise (LPN).

With constant error-rate τ, quantum runtime $2^{O(\frac{n}{\log(n/\tau)})}$ vs $2^{n/2}$ classical runtime.

Useful and desired features

Maybe the most useful feature while implementing:

Full cheap simulation of classical sub-circuits

- Q# exposes this via its "ToffoliSimulator", and allowed us to unit-test our circuits.
- I wrote a naive implementation for Qiskit for testing the AES S-box, at `https://github.com/fvirdia/Qiskit-DIY-Toffoli-simulator`.

Interesting functionality would be having a noise and error correction simulators.

Second most useful feature:

Language expressiveness

- Classical looping and branching notation, even conditioned on measurement output.
- Flexible qubit allocation (automatic and manual).
- Utilities such as “auto adjoint” computation.
- Both circuit-level and time-evolution descriptions.

Being able to compile to circuits is great.

Ok, I lied, this is the most useful feature:

Automatic resource estimation

- Including branches and measurements-conditioned components.
- Ideally with a mode for including error correction cost and imposing related constraints (like non-local qubit operations).

Something we definitely missed:

(Some) automatic circuit optimization

- Optimising qubit allocation (with min-depth vs min-width options).
- Automatic linear map circuits (with min-depth vs min-width vs min-gates options).
- "Quantum-exclusive" optimisations would be nice [GKMR14, ZC19].
- If an optimisation across scopes/function boundaries is found, report it clearly.

Something else we missed.

Documentation with examples

- We found lots of documentation consisted of function references without examples.
- I love the Qiskit textbook.

Ideally, this pairs with openly accessible bug tracking!

In Conclusion

- Cryptographers are adopting quantum programming languages.
- The use case presents different constraints.
- Some work also in the NISQ setting.

Thank you


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